Supernovae, the Equation of State, and Phase Transitions

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Abstract

The role of the equation of state (EOS) in supernova collapse is investigated. Improvements are made to the **BCK** parameterization, incorporating a new symmetry energy and prevention of superluminal behaviour, and first order phase transitions are included by specifying the location and width of the mixed phase region and high density limiting behaviour. The resulting EOS's make neutron stars sufficiently massive to satisfy observations and may be used to follow the evolution of a supernova through the formation of a neutron star. Numerical simulations using a new completely implicit hydrodynamical code are carried out. An unphysically soft EOS used with a small iron core leads to a direct explosion, while a stiff EOS falters with an accretion shock. The region near the accretion shock is remarkably similar in either case. Phase transitions are then included and have important consequences. The presence of a mixed phase region softens the EOS and leads to a direct explosion, with the EOS being sufficiently stiff to construct a neutron star compatible with observations. Due to the presence of electrons which do not participate in the phase transition, the phase change is muted and secondary shocks do not develop.

1 Introduction

When the center of a massive star with $M \gtrsim 10M_\odot$ exhausts its central fuel supply, it achieves nuclear statistical equilibrium with somewhat neutron rich iron peak nuclei. Subsequent contraction in search of a new fuel supply does not bear fruit and indeed a catastrophic implosion is the result. This implosion can only be stopped at densities in excess of nuclear

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matter density, $\rho_s \sim 0.16$ fm^{-3} $= 2.7 \times 10^{14}$ g cm^{-3} when the relative stiffness of nuclear matter reverses the implosion and launches a shock wave which then propagates outward through the star. This general scenario describes gravitational collapse supernovae which almost certainly account for Type II Supernovae, and perhaps Types Ib and Ic as well.

While the general fate of the iron core of such a massive star is inextricably tied to collapse, the specific mechanism of the supernova shock is not completely understood. It is now generally accepted that the simplest method known as the "direct shock" does not produce successful supernova explosions in numerical simulations when the best models of massive supernova progenitors are evolved through the explosive epoch using the best available physical ingredients. In a previous study [1]) we have shown that the direct, or prompt mechanism, will work only with iron cores which have attained no more than $\sim 1.18\ M_\odot$ when implosion runs away, a significantly lower mass that that found in recent presupernova evolutionary calculations. Furthermore these iron cores must also be quite "cold" as compared to such calculations. The second mechanism, the "delayed" mechanism originally invented by Wilson and his collaborators [2,3,4]), takes over when the prompt mechanism comes up short. A small fraction of the binding energy of the nascent neutron star, less than one tenth of a percent, which is escaping in neutrinos, manages to couple to the inner regions of the star through a variety of processes, leading to a revival of the shock wave and the eventual propagation through the rest of the star. While such a method may be more robust than the direct method, to date only the numerical simulations of the Wilson group have observed such explosions, and the viability of the method is strongly influenced by convective processes and how they are incorporated.

The physics of both mechanisms is strongly influenced by the equation of state (EOS) at high densities. The initial shock energy, the efficiency of energy transfer from the imploding material to the shock wave, is a direct function of the relative stiffness of the nuclear material. Indeed, before the importance of neutrino–electron scattering during the infall phase of the star was emphasized by Bruenn [5]) and its incorporation in the simulations produced dire consequences for the direct mechanism, the relative stiffness of the nuclear EOS was thought to be the controlling factor in the hydrodynamical simulations, with soft EOS's producing successful explosions, and stiff ones failures [6,7]).

Because the EOS of nuclear matter is not known with great certainty, simulations have employed phenomenological parameterizations which can be used to explore the sensitivity of supernovae to the EOS. A particularly useful form has been that of Baron, Cooperstein and Kahana [6,7])(**BCK**) which has been widely used (*cf.*, refs. 1, 8–11), and will be discussed in §2. Some improvements will be introduced into the **BCK** form, these being a more well–behaved symmetry energy and avoidance of superluminal behavior. In §3 neutron stars will be constructed using this improved EOS.

The **BCK** parameterization incorporates two essential nuclear properties; the bulk incompressiblity of nuclear matter for $\rho \sim \rho_s$, and the high density power law followed by the pressure. It is more than possible however, that the EOS may be more complex and may involve phase transitions of either first or second order. Such phase transitions have been proposed[12,13,14,15]) as the matter undergoes various condensations, such as to pion or kaon condensates, generally considered to be weak transitions, or to a deconfined phase of quark matter. Takahara and Sato [16]) have previously considered the effects of phase transitions on supernova collapse, using schematic equations of state and neutrino transport simplifications. In §4 we will construct EOS's including phase transitions, similar in spirit to those of Takahara and Sato [16]), but we will use them together with full neutrino transport methods and a detailed subnuclear equation of state.

Hydrodynamic infall through a fully first order phase transition can have dramatic effects. In a mixed phase region the pressure is constant and therefore the speed of sound vanishes. Thus one might expect an acceleration of the collapse and perhaps the development of a second shock wave as the material collapses past the mixed phase region. In §5 we investigate what happens when the star collapses through a phase transition and evaluate the ensuing effects on the supernova explosions. It turns out that due to the presence of electrons, the speed of sound never vanishes in the mixed phase region, and thus very dramatic behavior does not occur, although the effects on supernovae may be large anyway. In §6 we give some conclusions, and contrast with the work of Takahara and Sato [16]).

The hydrodynamical simulations of this paper have only been evolved for about 0.110 seconds after the bounce of the central core. The effects of possible phase transitions at later times, such as to a deconfined phase, will be considered in a paper under preparation.

2 EOS for $\rho > \rho_s$ without phase transitions.

For densities near the symmetric nuclear matter saturation density value, $\rho_s \sim 0.16$ fm^{-3}, the bulk energy per nucleon and the pressure at $T = 0$ may be written

$$\begin{aligned} E &= E_s + W_s\alpha^2 + \frac{K_s}{18}\left(\frac{\rho-\rho_s}{\rho_s}\right)^2 + \ldots \\ P &= \rho^2 \left.\frac{\partial E}{\partial \rho}\right|_\alpha \\ &= \frac{K_s\rho_s}{9}\left(\frac{\rho}{\rho_s}\right)^2\left(1-\frac{\rho}{\rho_s}\right) , \end{aligned} \tag{1}$$

where the neutron excess is given by

$$\alpha = \frac{N-Z}{A} \tag{2}$$

where $E_s = -16$ MeV W_s is the bulk symmetry energy coefficient and

$$K_s = 9 \left.\frac{\partial P}{\partial \rho}\right|_{\rho=\rho_s} \tag{3}$$

is the isospin symmetric nuclear bulk incompressibility. However, the results of microscopic calculations indicate that as the neutron excess α deviates from zero, isospin asymmetric matter still exhibits saturation, *i.e.*, the pressure should vanish at a density $\rho_s[\alpha] < \rho_s[0]$. Thus further terms in the expansion of Equation (1) are required; *i.e.*, the symmetry energy must be density dependent. To accomplish this **BCK** adopted the expansions

$$\begin{aligned} E &= E_s + W_s\alpha^2 + \frac{K_s[\alpha]}{18}(1-\theta)^2 \\ P &= \frac{K_s[\alpha]\rho_s[\alpha]}{9}\theta^2(1-\theta) \end{aligned} \tag{4}$$

where the bulk incompressibility is given a dependence on α as

$$K_s[\alpha] = K_s[0](1 - A\alpha^2) \tag{5}$$

and the density compression factor

$$\theta = \frac{\rho}{\rho_s[\alpha]} \tag{6}$$

depends on the saturation density as a function of α:

$$\rho_s[\alpha] = \rho_s[0](1 - B\alpha^2) \ , \tag{7}$$

with the coefficients A and B to be determined by microscopic calculations. **BCK** took $A = 2.0, \ B = 0.75$ from considering calculations with Skyrme forces with the SKM* force parameters [17]).

The expansion in Equation (4) is only valid in the neighborhood of ρ_s and is insufficient to specify the EOS at significantly higher densities such as are found in the center of collapsing stars or neutron stars. For higher densities **BCK** adopted the form

$$\begin{aligned} E &= E_s + W_s\alpha^2 + \frac{K_s[\alpha]}{9\gamma}\left(\frac{\theta^{\gamma-1} - 1}{\gamma - 1} + \frac{1}{\theta} - 1\right) \\ P &= \frac{K_s[\alpha]\rho_s[\alpha]}{9\gamma}\left(\theta^{\gamma} - 1\right) \end{aligned} \tag{8}$$

where the new parameter γ gives the high density behavior of the pressure. Near ρ_s Equation (8) reduces to Equation (4). The **BCK** parameterization of the EOS has been used in numerous investigations of supernovae [1,6–11]), the birth of neutron stars [18]) and has also been used to study relativistic heavy ion collisions [19]). This is because it is easy to use and contains as few parameters as possible while still containing a description of the EOS near and well above saturation density. The results of microscopic calculations can easily be fit to the **BCK** form.

However, the density dependence of the symmetry energy obtained from Equation (8) contains a serious deficiency, which can be seen by considering the chemical potential difference

$$\hat{\mu} = \mu_n - \mu_p = - \left.\frac{\partial E}{\partial x}\right|_{\rho} = 4 \left.\frac{\partial E}{\partial \alpha^2}\right|_{\rho} \tag{9}$$

which for the **BCK** Equation (8) becomes

$$\hat{\mu} = 4\alpha\left(W_s + \frac{d\ln K_s}{d\alpha^2}(E - E_s - W_s\alpha^2) - \frac{d\ln \rho_s}{d\alpha^2}\frac{P}{\rho}\right) \ . \tag{10}$$

For typical choices **BCK** used in hydrodynamical simulations, $K_s[0] = 180$, $\gamma = 2.5$, the decrease of K_s won out over the decrease of ρ_s (with α fixed and the density increasing) until the compressional contribution to $\hat{\mu}$ became negative; in fact at $\theta \sim 4$, $\hat{\mu}$ vanishes. This had two deleterious consequences. The first was that as $\hat{\mu}$ dropped electron captures increased because in β–equilibrium the relation $\hat{\mu} = \mu_e - \mu_\nu$ drove the matter to an increasing proportion of neutrinos at fixed lepton number. This considerably softened the EOS at bounce. Secondly, such a poorly behaved symmetry energy made it impossible to construct an EOS that led to neutron stars of sufficient mass to satisfy observations.

An improvement to the **BCK** EOS, to be denoted as **BCK′** will now be introduced. One writes the bulk energy per nucleon as

$$E = E_s + E_{\rm comp}[\theta] + \alpha^2 E_{\rm sym}[\theta] \tag{11}$$

where the redfined density compression factor

$$\theta \equiv \frac{\rho}{\rho_s} \tag{12}$$

is now independent of α and from now on $\rho_s \equiv \rho_s[0]$ and $K_s \equiv K_s[0]$, the symmetric nuclear matter values. With this decomposition one finds

$$\begin{aligned} P = &\ \theta^2 \rho_s \left.\frac{\partial E}{\partial \theta}\right|_\alpha &= &\ P_{\rm comp}[\theta] + \alpha^2 P_{\rm sym}[\theta] \\ K = &\ \frac{9}{\rho_s} \left.\frac{\partial P}{\partial \theta}\right|_\alpha &= &\ K_{\rm comp}[\theta] + \alpha^2 K_{\rm sym}[\theta] \ . \end{aligned} \tag{13}$$

It now remains to specify the compressional energy $E_{\rm comp}$, and the density dependent symmetry energy $E_{\rm sym}$. For the $E_{\rm comp}$ the original **BCK** form is used, but with the redefined θ, i.e.;

$\theta < 1$

$$\begin{aligned} E_{\rm comp} &= \frac{K_s}{18}(\theta - 1)^2 \\ P_{\rm comp} &= \frac{K_s \rho_s}{9} \theta^2 (\theta - 1) \\ K_{\rm comp} &= K_s(3\theta^2 - 2\theta) \end{aligned} \tag{14}$$

$\theta > 1$

$$
\begin{aligned}
E_{\text{comp}} = &\ \frac{K_s}{9\gamma}(\theta-1)^2\left(\frac{\theta^{\gamma-1}-1}{\gamma-1}+\frac{1}{\theta}-1\right) \\
P_{\text{comp}} = &\ \frac{K_s\rho_s}{9\gamma}\left(\theta^{\gamma}-1\right) \\
K_{\text{comp}} = &\ K_s\,\theta^{\gamma-1}\ .
\end{aligned}
\tag{15}
$$

With this construction,

$$
\hat{\mu} = 4\,\alpha\,E_{\text{sym}}[\theta]\ . \tag{16}
$$

The symmetry energy is constructed with the following properties in mind:

$$
\begin{aligned}
&1\ \ )\ \ \text{At } \theta=1, \quad E_{\text{sym}} = W_s \\
&2\ \ )\ \ P[\theta,\alpha]=0 \quad \text{when} \quad \theta = 1-B\alpha^2 \\
&3\ \ )\ \ K[\theta,\alpha]=K_s(1-A\alpha^2) \quad \text{when} \quad P=0\ .
\end{aligned}
\tag{17}
$$

While the above constraints, Equation (17), do not specify a unique EOS, one can make a choice which requires no new parameters (other than $E_s,\ K_s,\ W_s,\ A,\ B,\ \rho_s$):

$$
\begin{aligned}
E_{\text{sym}} &= W_s + \frac{AK_s}{9}\frac{(\theta-1)}{(1+\frac{A-2B}{2B}(\theta-1))} &&= W_s + \frac{K_s}{12}\frac{(\theta-1)}{(1+\frac{1}{3}(\theta-1))} \\
P_{\text{sym}} &= \frac{AK_s\rho_s}{9}\frac{\theta^2}{(1+\frac{A-2B}{2B}(\theta-1))^2} &&= \frac{K_s\rho_s}{12}\frac{\theta^2}{(1+\frac{1}{3}(\theta-1))^2} \\
K_{\text{sym}} &= (4B-A)K_s\frac{\theta}{(1+\frac{A-2B}{2B}(\theta-1))^3} &&= K_S\frac{\theta}{(1+\frac{1}{3}(\theta-1))^3}\ .
\end{aligned}
\tag{18}
$$

(The second form in these equations uses $A = 2.0$, $B = 0.75$.) This choice for E_{sym} has the behavior that $E_{\text{sym}} \sim \theta$, $P_{\text{sym}} \sim \theta^2$ for $\theta >> 1$, which is what one would expect from a symmetry energy arising from two–body meson exchange. However, we would like to reemphasize this choice of E_{sym} is not unique, and other symmetry energies with a different density dependence for $\theta >> 1$ could lead to quite different EOS's. This has been further discussed by Prakash, Ainsworth, and Lattimer [20]).

Figure 1 compares the **BCK** and **BCK′** EOS's, solid lines showing the new form and dashed lines the older one, for the representative case, $\gamma = 2.5,\ K_s = 180$ MeV, for $Y_e = 0.33$. The new EOS is seen to be somewhat stiffer at the same K_s and γ value, and $\hat{\mu}$ is seen to remain positive as the density increases.

One further improvement must be incorporated in the **BCK′** parameterization of the EOS. As was pointed out long ago by Bludman and Dover [21]), the dimensionless relativistic sound speed defined as

$$c_s \equiv \sqrt{\frac{dP}{d\varepsilon}} \tag{19}$$

where ε is the energy density (including the rest mass), should not exceed unity. If the EOS fixed by a particular choice of K_s and γ goes superluminal ($c_s > 1$) at some density θ_c, extrapolatation is undertaken for $\theta > \theta_c$ to obtain the unique form

$$\begin{aligned} E &= \frac{E_c}{2}\left(\frac{\theta}{\theta_c} + \frac{\theta_c}{\theta}\right) + \frac{P_c}{2\rho_s\theta_c}\left(\frac{\theta}{\theta_c} - \frac{\theta_c}{\theta}\right) \\ P &= \frac{\rho_s E_c}{2}\left(\frac{\theta}{\theta_c} - \frac{\theta_c}{\theta}\right) + \frac{P_c}{2}\left(\frac{\theta^2}{\theta_c^2} + 1\right) \ . \end{aligned} \tag{20}$$

independent of the form of the EOS for $\theta < \theta_c$. In the EOS's considered in this hydrodynamical study, superluminality never becomes a problem during the collapse of the star. However as the star proceeds to higher densities as the nascent neutron star forms, it is necessary for some of the EOS's to take Equation (20) into account.

A thermal contribution is added to the $T = 0$ EOS according to the prescription of Cooperstein and Baron [8]), involving an effective mass. This contribution is generally small in nuclear matter, and will not considered in detail here.

In addition to the baryonic EOS detailed above, the full EOS also includes the contributions of the electron–positron gas and photons. (Neutrinos are included through the transport algorithm summarized in §5.) The electrons and positrons are considered as free fermion gasses with finite mass, and their EOS is calculated accurately for all ranges of relativity and degeneracy. The photon radiation EOS is of the usual blackbody form.

3 Maximum neutron star masses.

For all EOS's employed in this study the general relativistic equations of stellar structure have been integrated to obtain the maximum mass as a function of central density, and with one exception (that used in Run 020), have been taken to satisfy $M_{\max} \gtrsim 1.5 M_\odot$. The relevant equations are

$$
\begin{aligned}
\frac{dP}{dR} &= -\frac{GM_G\varepsilon}{R^2}\frac{(1+4\pi R^3/M_G)(1+P/\varepsilon)}{(1-2GM_G/R)} \\
\frac{dM_G}{dR} &= 4\pi R^2\varepsilon \\
\frac{dM_B}{dR} &= \frac{4\pi R^2\rho}{(1-2GM_G/R)^{1/2}}
\end{aligned}
\tag{21}
$$

where M_G is the gravitational mass, M_B is the baryon mass, and we have taken $c = 1$. The binding energy is given by $-(M_G - M_B)c^2$. The above system of equations is closed by an EOS which gives P and ε as a function of the baryon density ρ. The mass of a neutron star for a given EOS is given by selecting the central density and integrating outward until the pressure vanishes. The maximum mass for a given EOS is obtained by varying the central density until the gravitational mass and binding energy are extremized. This procedure is mathematically equivalent to maximizing M_G at fixed M_B for simple EOS's such as those considered here.

Table 1 presents results for maximum neutron star masses for the EOS's used in the present study. Only the EOS in Run 020 fails to have a maximum mass in accord with observations; this run will be included for pedagogical purposes. Also included are results including β–equilibrium. Neutron stars are believed to have a significant proton fraction, with the number of electrons (ignoring muons) equal to the number of protons to respect charge neutrality. The proton fraction is determined by the condition

$$\mu_e = \hat{\mu} \tag{22}$$

and thus is a sensitive function of the symmetry energy. While the appearance of protons in neutron matter will soften the EOS, there arises also a significant pressure due to the electrons, and whether the total pressure is increased or decreased because of β–equilibrium is not predetermined [20,22]). If the neutron excess at equilibrium is denoted by $\alpha_{\rm eq}$ the difference in pressure between β–equilibrium matter and pure neutron matter will be given by

$$P[\alpha_{\rm eq},\theta] - P[1,\theta] = P_e - (1-\alpha_{\rm eq}^2)^2 P_{\rm sym}[\theta] \ . \tag{23}$$

The symmetry energy of Equation (18) produces $\alpha_{\rm eq} \sim 0.9$, or $Y_e \sim 0.05$. For symmetry energies which rise faster with density than Equation (18) the equilibrium fraction of protons increases and incorporating β–equilibrium may lead to a considerable softening of the EOS. However, in Table 1 the gravitational mass is hardly affected by inclusion of β–equilibrium for the EOS's studied, and even increases slightly in some cases.

It is worth emphasizing that while not very much is known about the EOS at high densities the requirements discussed above furnish significant constraints: avoiding $c_s > 1$ which gives an upper bound to stiffness; and requiring $M_{\rm max} > 1.5 M_\odot$ gives a lower bound to softness. Together with building in the known information at saturation density, the EOS must lie within specified boundaries.

4 EOS with phase transitions

We now construct EOS's exhibiting either first or second order phase transitions. We consider only the bulk energy at zero temperature, and suppose that the transition begins at a density ρ_A and ends at density ρ_B. In the mixed phase region extending from ρ_A to ρ_B, phase equilibrium requires constant pressure and enthalpy:

$$P_{\rm tr} = P_A = P_B, \qquad \mu_{\rm tr} = \mu_A = E_A + \frac{P_A}{\rho_A} = E_B + \frac{P_B}{\rho_B} = \mu_B \tag{24}$$

where $P_A = P[\rho_A]$, $E_A = E[\rho_A]$ and analogously for B, and $P_{\rm tr}$ and $\mu_{\rm tr}$ are the pressure and enthalpy in the mixed phase region. The energy discontinuity in the transition is given by

$$E_B - E_A = P_{\rm tr} \left(\frac{1}{\rho_A} - \frac{1}{\rho_B} \right) \ . \tag{25}$$

Within the mixed phase region a Maxwell construction gives the energy

$$E = \left(\frac{1/\rho - 1/\rho_B}{1/\rho_A - 1/\rho_B} \right) E_A + \left(\frac{1/\rho - 1/\rho_A}{1/\rho_A - 1/\rho_B} \right) E_B \ . \tag{26}$$

In the special case of ρ_A=ρ_B we have a second order phase transition and the energy discontinuity vanishes as well.

The A EOS will be furnished by the $\mathbf{BCK'}$ compressional expressions, Equation (15), for $\theta < \theta_A$ where $\theta_{A(B)} = \rho_{A(B)}/\rho_s$. For the B EOS the simplest form is that offered by a power law:

$$
\begin{aligned}
E_B &= E_{0B} + \frac{K_B \theta^{\gamma_B - 1}}{9\gamma_B(\gamma_B - 1)} \\
P_B &= \frac{K_B \rho_s \theta^{\gamma_B}}{9\gamma_B} \\
E_B + \frac{P_B}{\rho} &= \frac{K_B \theta^{\gamma_B - 1}}{9(\gamma_B - 1)} \ .
\end{aligned} \tag{27}
$$

Such a form would not work for an EOS which can not be represented by a simple power in the pressure, such as a bag model for a quark condensate which would require a constant term in the pressure.

Equation (27) requires specification of the three parameters K_B, E_{0B}, and γ_B. The method chosen is to first select θ_A and θ_B, *i.e.*, delineating the location and size of the mixed phase region, and then γ_B, giving the pressure's high density asymptotic behaviour. These conditions thus specify K_B and E_{0B} uniquely. The EOS is then made to avoid superluminality by invoking Equation (20) at high density, and is checked to ensure that the choice of parameters yields a maximum neutron star gravitational mass in excess of $1.5M_\odot$. To the compressional terms are added the same symmetry terms and thermal terms as for the A EOS.

It is assumed that the leptons (electrons and neutrinos) are not part of the phase transition, and no transition is put in either the symmetry energy or thermal energy. In principle the symmetry energy should be involved, but in choosing to parameterize the transition (by supplying θ_A, θ_B, γ_B), this can not be done without specifying a different form for the symmetry energy, as otherwise the phase transition equations are overdetermined. Since the neutron excess is relatively constant at high densities, this simplification does not matter much, but in general transitions involving the symmetry energy should indeed be included.

Two EOS's with phase transitions are shown in Figure 2. The upper figure has $K_0 = 240$ MeV, $\gamma = 3.0$, $\theta_A = 1.5$ and $\theta_B = 2.5$, and the lower figure has has $K_0 = 180$ MeV, $\gamma = 2.75$, $\theta_A = 2.0$ and $\theta_B = 2.5$. Both cases show what happens for $\gamma_B = 2.25$ to $\gamma_B = 3.75$, and the case with no phase transition. (The hydrodynamical runs will use $\gamma_B = 3.75$.) Displayed are the energy and pressure per nucleon without electrons (solid lines) and with electrons (dashed lines). (The figures take $\alpha = Y_e = 0.33$ and include symmetry energies as well.) The EOS with no phase transition is shown by long–dashed lines. Note that while a true mixed phase region with constant pressure appears when electrons are disregarded, the electron pressure continuously increases throughout the mixed phase region and so the phase transition is muted; the speed of sound does not vanish.

Results for maximum mass neutron stars are given in Table 1, which shows that the phase transition introduced in Run 023 decreases the mass, while that of Run 025 actually increases the mass. These particular phase transitions have been chosen for use in the hydrodynamical runs in §5 and their properties will be discussed when analysing the results.

5 Hydrodynamical Results

The present study utilizes a new hydrodynamical code whose purpose is to follow the evolution of a gravitational collapse supernova from the initial instability in the massive progenitor star, through the birth of the neutron star. Spherical symmetry is assumed within a fully general relativistic framework according to the procedure of Baron *et. al.* [23]). Fully implicit differencing is used for both the hydrodynamics and the neutrino transport, and thus timesteps are not restricted by the Courant stability condition. The pseudoviscosity method is used to calculate shocks. Dynamic rezoning is incorporated throughout the calculation in order to constantly supply fine zoning resolution in the vicinity of a shock wave and to remove zones where fine resolution is no longer required.

Neutrino transport is calculated using the "two–fluid" model of Cooperstein, van den Horn and Baron [24,25]). All types of neutrinos and antineutrinos are included and each is assumed to follow a Fermi-Dirac spectral distribution. The temperature and chemical potentials for each type are obtained by inverting the number and energy densities, whose evolution in time are independently calculated. Local thermodynamic equilibrium is never assumed and thus the neutrino temperatures are in principle different than the matter temperature. Rate equations are followed which include both lepton number and energy transfer terms between the matter and the neutrinos. The neutrino opacities are essentially those of ref. 24) with some minor updating and improvements incorporated. Flux limiting, using the method of Levermore and Pomraning [26]), is implemented according to the prescription of Baron *et. al.* [23]).

Full details of the code from which the present one has been developed are given in refs. 1) and 8). The major difference with the new code is the utilization of implicit hydrodynamics and the tuning of many details, as well as the new EOS employed.

The EOS for densities below nuclear matter density is essentially that given by Cooperstein [27]) and Cooperstein and Baron [8]), with some minor upgrading in the nuclear symmetry energy and partition function treatments. Together with incorporating the high density improvements described in this paper as **BCK′**, the result is an EOS capable of following the entire supernova process because it can cover material at all degrees of neutron richness and entropy at densities occured through supernova collapse and neutron star formation. In the outer regions of the core which have not achieved nuclear statistical equilibrium, nuclear burning is included in a simplified, flash–burning fashion. Photons are included and the electron and positron gas is calculated accurately for all levels of relativity and degeneracy.

The hydrodynamical runs take as their initial configuration the small iron core of Baron and Cooperstein [1]), with $M \sim 1.18 M_\odot$. The actual initial model is almost the same as Model 104 of ref. 1), minor alterations occuring only because of small differences in the EOS in the sub–nuclear regime. In each calculation about $1.5 M_\odot$ of the initial star is followed, including the iron core, the silicon shell and part of the oxygen shell, and it is divided into a number of Lagrangean mass zones varying from 75 to 120 according to the resolution requirements near the shock wave. Each simulation was followed about 100 milliseconds after bounce of the core.

Refs. 1) and 8) have summarized the main features of the early evolution of such hydrodynamic models, which have been studied by a number of authors. The inner regions of the iron core undergo a homologous collapse which is reversed due to the relative stiffness of nuclear matter, the core bouncing at a density several times greater than nuclear matter density. A shock wave forms near the sonic point at $R \sim 10$ km, $M \sim 0.7 M_\odot$, and then proceeds outwards. If the shock wave is strong enough it will continue to propagate outward through the mantle of the star, producing a "direct" explosion. In the case that it has insufficient energy to accomplish this, the shock wave will stall and form an accretion shock within a few tens of milliseconds. The subsequent fate of the accretion shock is not known with great certainty; the calculations of Wilson and his collaborators [2,3,4]) show that the shock can be revived on a time scale of a half second or more, by the absorption of a small fraction of the large neutrino flux escaping from the forming neutron star. The viability of the "delayed" mechanism, however, depends on the assumptions the authors make about how convection proceeds.

First we give a brief summary of the hydrodynamical runs to be presented. Run 020 uses a very soft EOS, producing a direct explosion, and Run 021 uses a very stiff EOS, producing a failed shock. Runs 022 and 024, using EOS's without phase transitions produce failures, while runs 023 and 025 use weak first order phase transitions and produce successful explosions. Important numerical results are summarized in Table 2 which gives the central density at bounce, the maximum radius the shock reaches in the calculation (or in the case of a successful explosion its position at the end of the calculation), the time after bounce, and the energy losses due to neutrinos exiting the computational grid at the end of the calculation. Figure 3 gives in encapsulated form, the results for the six runs presented, showing the evolution of the radius with time of Lagrangean mass points. Runs 020, 023 and 025 are seen to have shocks which reach to the edge of the computational grid when the calculation is terminated

5.1 Soft vs. Stiff EOS's: Runs 020 and 021

Run 020 uses a very soft EOS and Run 021 a very stiff one. From Table 2 we see that bounce is achieved at $\rho = 4.59\,\rho_s$ in the soft case, but only $\rho = 1.43\,\rho_s$ in the stiff case, and thus Run 020 begins with much more energy in the initial shock (**BCK**). Figures 4–6 show the hydrodynamical results for these two runs.

Figure 4 shows the propagation of the shock wave in these two runs and an important and general property is seen. In the case of the successful Run 020 the prompt shock moves directly out of the core while a secondary shock develops in the accretion region around the forming neutron star, just past 100 kilometers, about 0.020 seconds after bounce. In Run 021, the failure, both shocks also form, but the original prompt shock proceeds only to 180 kilometers before it falls back and joins up with the accretion shock. In Figure 5 other hydrodynamical properties are displayed at $t = 0.080$ seconds after bounce for each calculation, including the velocity, entropy, density, temperature, lepton number and electron fraction. In most respects the profiles are quite similar in the inner region near the accretion shock, except that the entropy is higher (and the density thereby somewhat lower) in the case of the successful shock. The region just past the shock wave looks somewhat unstable to convection due to the negative entropy gradient, but this is counterbalanced by the positive gradient in electron fraction, and only a numerical analysis can indicate how unstable this regime is with respect to convection.

The other information in Figures 4–6 give properties of the neutrinos. Figure 4 shows the development of the neutrino luminosities and average energy for each type of neutrino with time, as they exit the computational grid at several thousand kilometers. Electron neutrinos have a spike in luminosity as the shock leaves the neutrino sphere, the co-called neutronization burst, but then quickly settle down to a relatively constant luminosity. The electron antineutrino luminosity soon achieves the same value as that of the electron neutrinos, but the $\mu - \tau$ pairs remain at about half the value for each species. The reason for this can be seen in Figure 6 in which the luminosity is given with respect to radius at the end point of the calculation. The electron type neutrinos and antineutrino luminosity keeps increasing from $R \sim 35$ kilometers, near the neutrino sphere, while the pair neutrinos remain flat; the difference can thus be interpreted as the contribution to the neutrino luminosity due to accretion. As expected we find that $T_{\nu_e} < T_{\bar{\nu}_e} < T_{\nu_\mu}$ this being directly related to their relative opacities with the material. While some quantitative differences can be seen in the neutrino information at this time, the differences are probably too small to provide a direct signature about the mechanism from neutrino detection.

5.2 EOS with vs. without phase transitions: Runs 022 and 023, and Runs 024 and 025

Because Run 020 uses an unphysically soft EOS, too soft to construct a proper neutron star, we now proceed to look at stiffer EOS's and evaluate the role of phase transitions. Run 022 uses a stiff EOS with $\gamma = 3.00$ and $K_s = 240$ MeV and collapses to $\rho_b = 2.01\rho_s$ during the implosion phase. The shock wave proceeds outward to 250 kilometers before falling back and joining up with the accretion shock. In order to gauge the effects of possible phase transitions we now consider Run 023, where a phase transition begins at $\theta_A = 1.5$ and ends at $\theta_B = 2.5$, and then proceeds to $\gamma_B = 3.75$, chosen to provide a sufficient neutron star mass as shown in Table 1. This EOS is displayed in Figure 2. These parameter choices are made so that the phase transition is entered into just before bounce. We see from Table 2 that the center of the star now quickly collapses through the mixed phase region achieving $\rho_b = 4.08\rho_s$ before implosion is reversed.

Figures 7–9 give the same hydrodynamical information as given in Figures 4–6 for Runs 020 and 021, except that the Lagrangean mass is used as the abscissa in Figures 8 and 9, to get a somewhat different perspective. Looking at the entropy profiles displayed in Figure 8 we see that there is no change in the entropy in the inner core which traversed the mixed phase regime; no additional shock waves were induced by the fall through the low sound speed of the mixed phase region. The velocity of the shock wave appears small in the plots, but it is about 1000 km/sec at the end of the calculation.

Runs 024 and 025 are similar to those of the previous pair. They use a somewhat softer EOS to begin with, $\gamma = 2.75$ and $K_s = 180$ MeV and with no phase transition Run 024 collapses to $\rho_b = 2.461\,\rho_s$, and the shock wave makes it to 320 kilometers before falling back. Run 025 takes $\theta_A = 2.0$ and ends at $\theta_B = 2.5$, and then proceeds to $\gamma_B = 3.75$, as displayed in Figure 2. The center now collapses to $\rho_b = 3.40\,\rho_s$ before implosion is halted. The shock wave is somewhat weaker than that of Run 023, and is barely moving out at 700 kilometers when the calculation is terminated 0.105 seconds after bounce. Whether the shock wave can proceed further depends on the detailed composition of the outer burning layers, and whether sufficient energy is available from nuclear burning to revive the shock; since this is an artifical core, this can not really be analysed, so we will assume this is a marginal failure. The additional hydrodynamic details are quite similar to those of Runs 022 and 023 and are not presented for the sake of brevity.

6 Conclusions

An improved phenomenological parameterization of the EOS of dense matter has been constructed, appropriate to the thermodynamic regimes sampled in supernova collapse and neutron stars. The effects of hard and soft EOS's on the supernova mechanism have been considered together with the possible appearance of phase transitions in nuclear matter

It has been shown that first order phase transitions lying within a few times nuclear matter density may work exactly the same as a general softening of the EOS; no additional shock waves are born and no unusual hydrodynamic behaviour is observed. This is largely a consequence of having the leptons abstain from the phase transition, unlike the results of Takahara and Sato [16]) in which such secondary shock waves were seen. Takahara and Sato chose to parameterize the entire "cold" EOS, not including the electrons separately, with no explicit justification. To include the electrons properly, one must open up additional degrees of freedom in the symmetry energy and allow for differing charge and lepton fractions in the coexisting phases, a task for further investigation.

It is important to remark that the two runs with phase transitions, Runs 023 and 025 go briefly into the mixed phase region only during the bounce phase; they do not collapse back into the mixed phase region as the neutron star continues to contract until somewhat after the calculation has been terminated (about 0.1 seconds). Thus the general idea of **BCK**, that softening the EOS greatly strengthens the shock wave, is preserved. Improvements in the BCK symmetry energy of nuclear matter now permit EOS's which are both soft enough to strengthen the shock wave and stiff enough to build neutron stars.

The density reached in supernova collapse is now somewhat lower than was found by **BCK**, who found $\rho_b \sim 3.5\,\rho_s$ for the case $\gamma = 2.5$, $K_s = 180$ MeV. (**BCK** actually reported that the center bounced at 4.1 times nuclear matter density, but the nuclear matter density used was that for neutron rich matter, about 15 percent less.) For this case the **BCK′** would give $\rho_b \sim 2.5\,\rho_s$. This discrepancy arises partly from the fact that for the same values of γ and K_s **BCK′** is somewhat stiffer than **BCK**, as can be seen in Figure 1. However, the rapid softening of the symmetry energy in **BCK** caused the center of the star to collapse to a much higher density due to neutronization at the last moments of infall, which considerably softened the EOS.

The present study shows both prompt explosions and failed shocks generate strong accretion shocks outside the forming neutron star, and that other than a shock moving out at large radius in the prompt case, the material near and inside the inner accretion shock looks remarkably the same in either case. This implies that the physics which propels the delayed mechanism should operate essentially the same in either case; in the case of the prompt shock it would amplify the strength of the shock over the next second or so. Implications for nucleosynthesis in the hot supernova bubble should also apply equally well in either case.

At times greater than one tenth of a second after bounce, the core will continue to contract to higher densities, and further phase transitions may occur, such as that to a deconfined phase. Additionally, the center of the star will find itself deeply into the mixed regime of the phase transitions included in this study and this may have consequences on the neutrino signal and/or the shock mechanism. Development on these longer time scales is now under investigation and results will be reported.

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References

1. E. Baron and J. Cooperstein, *Astrophys. J.* **353** (1990) 597

2. J. R. Wilson, in *Relativistic Astrophysics,* J. Centrella and J. LeBlanc and R. Bowers, eds., (Boston: Jones and Bartlett) (1985) 422

3. H. A. Bethe and J. R. Wilson, J. R., *Astrophys. J.* **263** (1985) 366

4. R. Mayle, in *Supernovae,*, A. Petschek, ed. (New York: Springer) (1990) 267

5. S. Bruenn 1985, *Astrophys. J. Suppl.* **58** (1985) 771

6. E. Baron, J. Cooperstein, and S. Kahana, *Nucl. Phys.* **A440** (1985) 774

7. E. Baron, J. Cooperstein, and S. Kahana, *Phys. Rev. Lett.* **55** (1985) 126

8. J. Cooperstein and E. Baron, in *Supernovae,*, A. Petschek, ed. (New York: Springer) (1990) 213

9. S. Bruenn, *Astrophys. J.* **340** (1989) 955

10. S. Bruenn, *Astrophys. J.* **341** (1989) 385

11. E. Myra and S. A. Bludman, *Astrophys. J.* **340** (1989) 384

12. W. Weise and G. E. Brown, *Phys. Rept.* **27C** (1976) 1

13. G. Baym and S. A. Chin, *Phys. Lett.* **62B** (1976) 241

14. A. B. Migdal, *Rev. Mod. Phys.* **50** (1978) 107

15. R. Tamagaki, T. Tatsumi and I. Watanabe, *Prog. Theor. Phys.* **71** (1984) 569

16. M. Takahara and K. Sato, *Astrophys. J.* **335** (1988) 301; *Prog. Theor. Phys.* **68** (1982) 795; *Prog. Theor. Phys.* **71** (1984) 524; *Prog. Theor. Phys.* **72** (1984) 978; *Phys. Lett.* **156B** (1985) 17

17. K. Kolehmainen, M. Prakash, J. Lattimer, and J. Treiner, *Nucl Phys* **A439** (1985) 535

18. A. Burrows and J. M. Lattimer, *Astrophys. J.* **307** (1986) 187

19. T. Ainsworth, E. Baron, G. E. Brown, J. Cooperstein, and M. Prakash, *Nucl Phys* **464** (1987) 740

20. M. Prakash, T. Ainsworth, and J. M. Lattimer, *Phys. Rev. Lett.* **61** (1988) 2518

21. S. A. Bludman and C. B. Dover, *Phys. Rev.* , **D22** (1980) 1333

22. J. Cooperstein, *Phys. Rev.* **C37** (1988) 786

23. E. Baron, E. S. Myra, J. Cooperstein, and L. J. van den Horn, *Astrophys. J.* **339** (1989) 978

24. J. Cooperstein, L. J. van den Horn, and E. Baron, *Astrophys. J.* **309** (1986) 653; erratum, *Astrophys. J.* **315** (1987) 719

25. J. Cooperstein, L. J. van den Horn, and E. Baron, *Astrophys. J. Let* **321** (1987) L129

26. C. D. Levermore and G. C. Pomraning, *Astrophys. J.* **248** (1981) 321

27. J. Cooperstein, *Nucl. Phys.* **A438** (1985) 722

Tables

TABLE 1. **TOV Results**

						with β–eq.			without β–eq.		
RUN	γ	K_0	θ_A	θ_B	γ_B	M_G	M_B	ρ_c/ρ_s	M_G	M_B	ρ_c/ρ_s
020	2.00	140	...	...	...	0.725	0.781	33.00	0.722	0.775	34.37
021	4.00	400	...	...	...	2.791	3.485	4.67	2.791	3.465	4.69
022	3.00	240	...	...	...	1.879	2.223	8.95	1.876	2.209	8.95
023	3.00	240	1.50	2.50	3.75	1.494	1.775	14.32	1.488	1.759	14.26
024	2.75	180	...	...	...	1.535	1.782	12.66	1.535	1.782	12.66
025	2.75	180	2.00	2.50	3.75	1.701	2.044	11.42	1.701	2.044	11.42

Properties of maximum mass neutron stars with and without incorporation of β–equilibrium. The gravitational mass M_G and the baryon mass M_B are given in solar masses, K_0 in MeV. ρ_c/ρ_s is the central density in units of nuclear matter density.

TABLE 2. **Hydrodynamical results**

RUN	$\rho_b(14)$	ρ_b/ρ_0	$R_S^{\max}$	Exp.?	$t - t_b$	E_{ν_e}	$E_{\bar{\nu}_e}$	E_{ν_μ}	ALL
020	12.20	4.59	>2500	Yes	0.119	8.47	6.08	10.00	24.65
021	3.80	1.43	180	No	0.079	6.35	3.52	6.77	16.64
022	5.35	2.01	250	No	0.108	7.95	5.18	9.66	22.79
023	10.83	4.08	1000	Yes	0.116	8.53	5.92	9.45	23.90
024	6.53	2.46	400	No	0.104	7.94	5.22	9.40	22.58
025	9.06	3.40	>700	?	0.104	8.03	5.38	8.68	22.09

EOS parameters are given in Table 1. ρ_b is the central density at bounce, in units of 10^{14} g cm^{-3} and $\rho_s = 0.16$ fm^{-3}. $R_S^{\max}$ is the maximum shock radius in kilometers. Exp. indicates whether or not a successful explosion is the result. $t - t_b$ is the time after bounce (in seconds) when the calculation is stopped. E_{ν_e}, $E_{\bar{\nu}_e}$, and E_{ν_μ}, are the total losses in energy due to neutrinos in units of 10^{51} ergs at the end of the calculation. ALL $= E_{\nu_e} + E_{\bar{\nu}_e} + E_{\nu_\mu}$ in the same units.

Figures

FIG. 1. The **BCK** and **BCK′** EOS's are denoted by solid and dashed lines respectively. The case $\gamma = 2.5$, $K_s = 180$ MeV, $Y_e = 0.33$ is given. Shown as a function of $\theta = \rho/\rho_s$ are: $\hat{\mu}$, μ_n in MeV, P in MeV fm^{-3} and E in MeV.

FIG. 2. EOS with phase transitions. The pressure P is shown in MeV fm^{-3}, and the energy per nucleon E in MeV, as a function of the density $\theta = \rho/\rho_s$, in units of nuclear matter density $\rho_s = 0.16$ fm^{-3}. The long–dashed lines shows the EOS with no phase transition. The upper figures have $K_0 = 240$ MeV, γ=3.00 and exhibit phase transitions extending from $\theta_A = 1.5$ to $\theta_B = 2.5$; the lower figures have $K_0 = 180$ MeV, γ=2.75 and exhibit phase transitions extending from $\theta_A = 2.0$ to $\theta_B = 2.5$. Solid lines show the EOS with phase transitions to $\gamma_B = 2.25$, 3.00, 3.75, higher values lying higher in the graphs. The EOS's used in the hydrodynamical runs have $\gamma_B = 3.75$. Short–dashed lines show the EOS's when electrons are included. ($x = 0.33$ has been assumed.) The mixed phase region no longer has constant pressure.

FIG. 3. Hydrodynamical results. For each hydrodynamical run, contours of constant Lagrangean mass point are shown in the $\log_{10} R - (t - t_b)$ plane, with R in centimeters, and $t - t_b$ giving the time since bounce in seconds. Each line follows a given mass point with solid lines giving $M = 0.1, 0.2, \ldots 1.4\, M_\odot$ and the dashed lines giving mid values. In Runs 020, 023, and 025 the shock moves to large radii in the outer core while in Runs 021, 022, and 024 the shock falls back into accretion at $R \sim 100$ km.

FIG. 4. Propagation of the shock wave for Runs 020 and 021. The top two figures show the position of the shock wave as a function of $t - t_b$, the time after bounce in seconds. The top figure gives the position in $\log_{10} R$, the second in $M/M_\odot$. Each point displayed shows a zone in which the pseudoviscosity is more than 10 percent of the pressure, indicating the range of the shock wave(s). The third figure from the top gives the L_{51}, the luminosity in neutrinos at the edge of the computational grid Qin units of 10^{51} erg s^{-1}. The short–dashed line gives the contribution of ν_e, the long–dashed line that of $\bar{\nu}_e$

and the dashed–dotted line that of each species of $\nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau}$, and the solid line gives the sum of all types. The fourth figure gives the average neutrino energy for each type of neutrino as it leaves the grid (in MeV), with the same line convention as for the previous figure.

FIG. 5. Profiles of Run 020 (solid lines) and Run 021 (dashed lines) at 0.080 seconds after bounce. As a function of $\log_{10} R$ are shown: v_9, the velocity in units of 10^9 cm s^{-1}; S the entropy per nucleon; $\log_{10} \rho$ the cgs density; T the matter temperature in MeV; Y_l the number of leptons per nucleon; and Y_e the number of electrons minus positrons per nucleon.

FIG. 6. Neutrino profiles for Runs 020 and 021 at $t - t_b = 0.080$ seconds after bounce. Shown are the neutrino temperatures T_ν and the average neutrino energy E_ν in MeV as a function of radial position, and L_{51}, the luminosity in neutrinos in units of 10^{51} erg s^{-1}. In each graph, the short–dashed line denotes ν_e, the long–dashed line denotes $\bar{\nu}_e$ and the dashed–dotted line denotes each species of $\nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau}$. In the T_ν plot the solid line shows the matter temperature and in the L_{51} plot the solid line shows the total luminosity in neutrinos.

FIG. 7. Same as Figure 4, except for Runs 022 and 023.

FIG. 8. Same as Figure 5, except for Run 022 (solid lines) and Run 023 (dashed lines), at $t - t_b = 0.109$ seconds, and the Lagrangean mass coordinate is used for the abscissa in units of $M_\odot$.

FIG. 9. Same as Figure 6, except for Run 022 (solid lines) and Run 023 (dashed lines), at $t - t_b = 0.109$ seconds, and the Lagrangean mass coordinate is used for the abscissa in units of $M_\odot$.