Supernova Mechanisms:
The first second after bounce, the equation of state and neutrino fluxes.

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Abstract. New hydrodynamical calculations of gravitational collapse supernovae are presented. Whether or not a direct explosion results, a strong accretion shock develops at the surface of the forming neutron star. Soft and stiff EOS's are considered. First order phase transitions are seen to soften the EOS and strengthen the shock wave. Temperatures as high as $T = 75$ MeV are obtained and the nuclear matter is not degenerate. Free pions are included in the EOS.

1. Introduction

Supernovae of Type II (and probably Type Ib) are generally believed to be the visible displays of the death of massive stars through explosion. When the central region of these stars has burnt all the way to a nuclear statistical equilibrium distribution of the iron peak elements, no further fusion reactions can liberate energy for the further support of the overlying massive star. Catastrophic gravitational implosion ensues until the center achieves densities several times that found in nuclei, at which point the core rebounds and launches a shock wave. Somewhow this shock wave then propagates through the overlying material, eventually bursts through the surface, and a supernova explosion is accomplished. However, after almost three decades of attempts at numerical simulation of this process, a clear picture of exactly how the mechanism works has yet to emerge. Because several other talks at this symposium have given a detailed overview of this situation, I will move directly into discussion of several aspects which have been considered in the present study.

I will present a brief progress report on simulations developed with a completely new implicit hydrodynamical code. The subjects addressed will be: use of an improved symmetry energy for neutron rich matter; the effects of a soft and a stiff equation of state (EOS); the consequences of first order phase transitions in nuclear matter; the neutrino fluxes obtained during the first second after bounce of the central core; the behaviour

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of the material in the region of the phase diagram near the critical point for the liquid–vapor phase transition, and improvements in the thermal EOS in this region; and the effects of inclusion of negative pions.

2. Calculation ingredients and an overview of hydrodynamical runs

2.1. Hydrodynamics

The hydrodynamical code is a completely new one, using the fully general relativistic formalism of Baron *et al* [1], is one dimensional and assumes spherical symmetry. The differencing is done in a completely implicit manner, which allows timesteps far in excess of the Courant stability criteria, which would restrict Δt to be less than the sound travel time across a zone. (Speed up factors of up to more than one hundred have been achieved when the shock is stalled in accretion, twenty milliseconds or so after bounce. These savings directly translate into CPU time.) Dynamic rezoning is employed to split zones when resolution is required, and to combine them when it is not. A typical calculation has about 100 Lagrangean zones, and may experience up to one thousand rezonings, giving fine resolution around the shock front, which is simulated through the standard method of pseudoviscosity.

2.2. Equation of state (EOS)

For densities $\rho < \rho_s$, the baryonic material is described by a four component nuclear statistical equilibrium network, consisting of free neutrons and protons, alpha particles, and a representative heavy nucleus whose properties are given by the compressible liquid drop model, according to the fashion employed in previous work [2, 3]. For $\rho > \rho_s$ the parametrization of Baron, Cooperstein and Kahana (**BCK**) [4, 5] has been adopted with improvements to the symmetry energy to be detailed in §3. The benchmark EOS has also been generalized to include first order phase transitions in nuclear matter as will be described in detail elsewhere [6] and summarized in §5. Thermal energies will be improved to account for all states of degeneracy in §6, and to include pions in §8. Also included are electrons and positrons (done exactly for all degrees of degeneracy and relativity) and photons.

2.3. Neutrino transport

The flow of neutrinos is calculated within the framework of the two fluid model [7, 8, 9], in which it is assumed that the neutrino distribution is of the fermi–dirac form, but with temperature and chemical potential different from that of the matter; *i.e.*, the neutrinos are not in local thermodynamic equilibrium with respect to the matter. All types of neutrinos and antineutrinos are included, and local source and scattering terms are calculated. The transition between diffusion and free streaming is calculated using the technique of flux–limiting, using a variation of the method of Levermore and Pomraning [10] as detailed in Cooperstein and Baron [9]. All neutrino diffusion calculations are done using implicit differencing schemes and there is no operator split with the matter variables.

2.4. Initial models

Two different initial models have been used for the initial stages of collapse. One model has been furnished by Weaver and Woosley [11] and corresponds to a 20 $M_\odot$ star, evolved using restricted semi–convection and a high rate for the reaction $^{12}C(\alpha,\gamma)^{16}O$. The other model uses the small iron core of Baron and Cooperstein [12], with $M \sim 1.18\ M_\odot$. The Weaver–Woosley model is used to look at long time behaviour in the accretion shock, and the Baron and Cooperstein model to look at the effect of the EOS on the initial shock wave.

2.5. Maximum neutron star masses.

For all EOS's employed in this study the general relativistic equations of stellar structure have been integrated to obtain the maximum mass as a function of central density, and with two exception (those used in Runs 020 and 039–040), have been taken to satisfy to have a maximum $M_G > 1.5\ M_\odot$. (M_G is the gravitational mass and M_B is the baryon mass.) For a given EOS one integrates outward until the pressure vanishes, varying the central density until the gravitational mass, and hence the binding energy, are extremized. thereby obtaining the maximum mass. Table 1 shows results for maximum neutron star masses for the EOS's used in the present study. The EOS in Run 020 with a very low maximum mass is included to show the hydrodynamical impact of a very soft EOS while the slightly too soft EOS of Run 039 is considered in order to see when a black hole might result. Also included are results including β–equilibrium. (Neutron stars exhibit a significant proton fraction, with the number of electrons equal to the number of protons to respect charge neutrality.) For the symmetry energy used in this study, masses are hardly altered.

2.6. Overview of hydrodynamical results

Twelve hydrodynamical simulations have been carried out and their properties are summarized in Table 2. The Weaver–Woosley 20 $M_\odot$ model [11] is used in Runs 033, 038–041, and the Baron and Cooperstein 1.18 $M_\odot$ model [12] is used in Runs 020–025, 042. Runs 020 and 021 compare the effects of a soft and stiff EOS. Runs 022 and 023 offer a stiff EOS with and without a phase transition, and Runs 024 and 025 the same for a softer EOS. Runs 033, 038 and 041 uses a stiff EOS, Run 038 differing from Run 033 in the use of an improved thermal energy and it has been run the longest, for 1.1 seconds after bounce. Run 041 differs from from Run 038 by the inclusion of pions. Run 039 and Run 040 have a soft EOS and improved thermal energy with Run 040 including pions. Run 042 is the same as Runs 040 and 041 except it uses an EOS intermediate in stiffness. Runs 020 and 023 give successful direct explosions, and Run 025 gives a marginal one. Runs 039 and 040 collapse into black holes.

3. EOS for $\rho > \rho_s$ and the symmetry energy

I now constuct an improvement to the **BCK** parameterization of the EOS, to be denoted as **BCK′**. Defining the density in units of ρ_s as $\theta \equiv \rho/\rho_s$ and the neutron excess by

Table 1. Properties of maximum mass neutron stars with and without incorporation of β–equilibrium. M_G = Gravitational Mass (in $M_\odot$), M_B = Baryon Mass (in $M_\odot$), K_0 in MeV, ρ_c/ρ_s = central density in units of nuclear matter density. The phase transition parameters θ_A, θ_B, and γ_B are considered in §5.

						with β–eq.			without β–eq.		
RUN	γ	K_0	θ_A	θ_B	γ_B	M_G	M_B	ρ_c/ρ_s	M_G	M_B	ρ_c/ρ_s
020	2.00	140	...	...	...	0.725	0.781	33.00	0.722	0.775	34.37
021	4.00	400	...	...	...	2.791	3.485	4.67	2.791	3.465	4.69
022	3.00	240	...	...	...	1.879	2.223	8.95	1.876	2.209	8.95
033	3.00	240	...	...	...	”	”	”	”	”	”
038	3.00	240	...	...	...	”	”	”	”	”	”
041	3.00	240	...	...	...	”	”	”	”	”	”
023	3.00	240	1.50	2.50	3.75	1.494	1.775	14.32	1.488	1.759	14.26
024	2.75	180	...	...	...	1.535	1.782	12.66	1.535	1.782	12.66
042	2.75	180	...	...	...	“	”	”	”	”	”
025	2.75	180	2.00	2.50	3.75	1.701	2.044	11.42	1.701	2.044	11.42
039	2.50	180	...	...	...	1.313	1.492	16.00	1.309	1.482	15.89
040	2.50	180	...	...	...	”	”	”	”	”	”

$\alpha \equiv (N-Z)/A$ one writes the bulk energy and pressure of nuclear matter as

$$E = E_s + E_{\rm comp}[\theta] + \alpha^2 E_{\rm sym}[\theta] \qquad (1)$$

$$P = \theta^2 \rho_s \left.\frac{\partial E}{\partial \theta}\right|_\alpha = P_{\rm comp}[\theta] + \alpha^2 P_{\rm sym}[\theta] \qquad (2)$$

where $E_s = -16$ MeV and $\rho_s = 0.16$ fm^{-3}. The compressional terms are given by

$\theta < 1$

$$E_{\rm comp} = \frac{K_s}{18}(\theta-1)^2 \ , \qquad P_{\rm comp} = \frac{K_s \rho_s}{9}\theta^2(\theta-1) \qquad (3)$$

$\theta > 1$

$$E_{\rm comp} = \frac{K_s}{9\gamma}(\theta-1)^2\left(\frac{\theta^{\gamma-1}-1}{\gamma-1} + \frac{1}{\theta} - 1\right) \ , \qquad P_{\rm comp} = \frac{K_s\rho_s}{9\gamma}\left(\theta^\gamma - 1\right) \ . \qquad (4)$$

These differ from the original **BCK** [4, 5] expressions in their use of the ρ_s in the definition of θ rather than the saturation density of neutron rich matter. For the symmetry energy I use

$$E_{\rm sym} = W_s + \frac{K_s}{12}\frac{(\theta-1)}{(1+\frac{1}{3}(u-1))} \ , \qquad P_{\rm sym} = \frac{K_s\rho_s}{12}\frac{\theta^2}{(1+\frac{1}{3}(u-1))^2} \ . \qquad (5)$$

The phenomenological parameters are thus E_s, ρ_s, W_s the bulk symmetry energy, K_s the nuclear bulk incompressibility which determines the stiffness of the EOS near ρ_s, and γ, which gives the stiffness at high density. This choice of symmetry energy has been constructed with the following properties in mind [6]: at $\theta = 1$ one obtains $E_{\rm sym} = W_s$ the

Table 2. Hydrodynamical results. ρ_b is the central density at bounce, in units of 10^{14} g cm^{-3} and $\rho_s = 0.16$ fm^{-3}. $R_S^{\max}$ is the maximum shock radius in kilometers. Exp. indicates whether or not a successful explosion is the result. π^-? indicates whether π^- are included. E^*? indicates whether improved thermal energies are included. $t - t_b$ is the time after bounce (in seconds) when the calculation is stopped. E_{ν_e}, $E_{\bar{\nu}_e}$, and E_{ν_μ}, are the total losses in energy due to neutrinos in units of 10^{51} ergs at the end of the calculation. ALL $= E_{\nu_e} + E_{\bar{\nu}_e} + E_{\nu_\mu}$ in the same units.

RUN	$\rho_b(14)$	ρ_b/ρ_0	$R_S^{\max}$	Exp.?	π^-?	E^*?	$t - t_b$	E_{ν_e}	$E_{\bar{\nu}_e}$	E_{ν_μ}	ALL
020	12.20	4.59	>2500	Yes	No	No	0.119	8.47	6.08	10.00	24.65
021	3.80	1.43	180	No	No	No	0.079	6.35	3.52	6.77	16.64
022	5.35	2.01	250	No	No	No	0.108	7.95	5.18	9.66	22.79
023	10.83	4.08	1000	Yes	No	No	0.116	8.53	5.92	9.45	23.90
024	6.53	2.46	400	No	No	No	0.104	7.94	5.22	9.40	22.58
025	9.06	3.40	>700	?	No	No	0.104	8.03	5.38	8.68	22.09
033	5.18	1.95	110	No	No	No	0.378				63.86
038	4.93	1.85	150	No	No	Yes	1.100	56.55	49.75	59.27	166.03
039	6.28	2.36	150	No	No	Yes	0.420	23.54	19.52	33.78	76.38
040	6.73	2.53	150	No	Yes	Yes	0.323	17.71	13.74	27.03	58.22
041	5.13	1.93	150	No	Yes	Yes	0.594	33.78	28.11	40.94	102.83
042	6.57	2.47	165	No	Yes	Yes	0.819	35.66	30.82	32.90	99.58

bulk symmetry coefficient; both the saturation density and the nuclear incompressibility drop quadratically with neutron excess with coefficients determined by the microscopic calculations of Kolehmainen *et al* [13].

4. Soft and stiff equations of state

Run 020 uses a very soft EOS, given by the parameters $\gamma = 2$, $K_s = 140$ MeV. As shown in Table 1 this EOS is unphysical as it can not produce a physical neutron star. Run 021 uses a very stiff EOS with $\gamma = 4$, $K_s = 400$ MeV. Figure 1 shows the position of the shock for the first 100 milliseconds after bounce in these two cases. In each case a double shock is seen. The first shock is launched with the rebounding core and in the case of Run 020 succeeds in reaching the outer parts of the star without delay. In Run 021 the shock is not strong enough to propagate through the overlying material and falls back in 30 milliseconds to join up with a secondary accretion shock, which develops in each case at about 20 milliseconds after bounce. As shown in Table 2, the shock never passes 180 km in Run 021; it joins the accretion shock at $R \sim 110$ km. A detailed look at snapshots at the end of the calculations would show that the two runs look surprisingly the same, except for the presence of the outer shock in the successful Run 020. The physics at the accretion front looks very much the same in either case, at the surface where material is raining in on the forming neutron star.

Figure 1. Position of the shock for Runs 020 (soft EOS) and 021 (stiff EOS). The abscissa is the time after bounce in seconds and the ordinate is the $\log_{10}(R)$, the radius in centimeters.

5. Phase transitions in nuclear matter

Various phase transitions are believed to go on in supernova matter. Below nuclear matter density studies have been made of liquid–vapor phase coexistance, the nuclei–bubbles transition at $\rho \sim \frac{1}{2}\rho_s$ and the bubbles–nuclear matter transition at $\rho \sim \frac{4}{5}\rho_s$. Above ρ_s there may be first and second order phase transitions to various condensates such as pion, kaon, or deconfined quark matter. This study considers what happens if there is a first order phase transition at several times nuclear matter density.

I assume that the phase transition involves only the baryonic component, *i.e.*, the electrons do not participate, and consider it taking place only for the $T = 0$ compressional terms. Taking the transition as beginning at ρ_A and ending at ρ_B, the material is in a mixed phase from ρ_A to ρ_B, in which there is constant pressure and enthalpy, $P_{\rm tr}$ and $\mu_{\rm tr}$:

$$P_{\rm tr} = P_A = P_B, \qquad \mu_{\rm tr} = \mu_A = E_A + \frac{P_A}{\rho_A} = E_B + \frac{P_B}{\rho_B} = \mu_B \tag{6}$$

where

$$P_{A,B} = P[\rho_{A,B}], \qquad E_{A,B} = E[\rho_{A,B}] \; . \tag{7}$$

In the first order phase transition there is an energy discontinuity given by

$$E_B - E_A = P_{\rm tr}\left(\frac{1}{\rho_A} - \frac{1}{\rho_B}\right) \; , \tag{8}$$

and within the mixed phase region a Maxwell construction gives

$$E = \left(\frac{1/\rho - 1/\rho_B}{1/\rho_A - 1/\rho_B}\right) E_A + \left(\frac{1/\rho - 1/\rho_A}{1/\rho_A - 1/\rho_B}\right) E_B \; . \tag{9}$$

When ρ_A=ρ_B , the phase transition collapses to second order. For the A EOS, the **BCK′** compressional EOS is used, and for the B EOS the simplest form is a power law:

$$\begin{aligned} E_B &= E_{0B} + \frac{K_B\theta^{\gamma_B-1}}{9\gamma_B(\gamma_B-1)} \; , \qquad P_B = \frac{K_B\rho_s\theta^{\gamma_B}}{9\gamma_B} \\ E_B + \frac{P_B}{\rho} &= \frac{K_B\theta^{\gamma_B-1}}{9(\gamma_B-1)} \; . \end{aligned} \tag{10}$$

Thus once the parameters of the A EOS are known, one must specify K_B, E_{0B}, and γ_B. The procedure taken is to select ρ_A and ρ_B, (giving the location and size of mixed phase) and γ_B (giving the high density behaviour), which then trivially determine K_B and E_{0B} uniquely.

Figure 2. EOS with phase transitions. The pressure P is shown in MeV fm^{-3}, and the energy per nucleon E in MeV, as a function of the density $u = \rho/\rho_s$, Long–dashed lines show the EOS with no phase transition. The upper figures have $K_0 = 240$ MeV, γ=3.00 and exhibit phase transitions extending from $\theta_A = 1.5$ to $\theta_B = 2.5$; the lower figures have $K_0 = 180$ MeV, γ=2.75 and take $\theta_A = 2.0$, $\theta_B = 2.5$. Solid lines show the EOS with phase transitions to $\gamma_B = 2.25$, 3.00, 3.75, higher values lying higher in the graphs. The EOS's used in Runs 023 and 025 have $\gamma_B = 3.75$. Short–dashed lines show the EOS's when electrons are included. ($\alpha = 0.33$ has been assumed.)

Figure 2 shows the EOS's with and without phase transitions used in Runs 022–025. Because Run 022(024) bounces at $\theta = 2.01(2.46)$ the phase transition in Run 023 was chosen to start just before this, at $\theta = 1.5(2.0)$. When electrons are included (short–dashed lines) the mixed phase region no longer has constant pressure, which has important hydrodynamical consequences.

When a fully first order phase transition occurs, the sound speed, given relativistically as $c_s = \sqrt{\frac{\partial p}{\partial \epsilon}}$, where ϵ is the energy density, vanishes in the mixed phase region. Thus as hydrodynamic infall proceeds through such a regime a the flow must become supersonic and a shock wave must form.

Run 022 uses a stiff EOS, $K_s = 240$ MeV and $\gamma = 3.0$, and without a phase

Figure 3. Position of the shock for Runs 022 (stiff EOS) and 023 (stiff EOS with phase transition) . The abscissa is the time after bounce in seconds and the ordinate is the $\log_{10}(R)$, the radius in centimeters.

transition bounces at $\rho = 2.01\ \rho_s$. The shock wave reaches only 250 km before falling back. Run 023 includes a first order phase transition and collapses to a much higher density, $\rho = 4.08\ \rho_s$ before rebounding. The shock wave is much stronger and a successful explosion is the result. Figure 3 shows the position of the shock wave in these two runs, and the pattern is just the same as seen in the soft–stiff comparison of Runs 020 and 021. To look in some more detail figure 4 shows the profiles at 110 milliseconds after bounce Except for the somewhat higher density at the center and the blip in outward moving velocity in Run 023, the two runs look almost exactly the same. So the general effect is to simply stengthen the shock wave by softening the EOS. However, it is important to point out that the EOS of Run 023 is sufficiently stiff to generate a maximum mass neutron star of $M = 1.494\ M_\odot$ (with β–equilibrium). Runs 024 and 025 are simlilar as a pair to Runs 022 and 023, except that the phase transition starts a little later and the explosion is weaker.

6. Thermal energies and the critical point: Accretion vs. Cooling and T in the First Second

Initially, material in the nuclear matter regime has $S \lesssim 1.5$ because the shock forms at the sonic point, $M \sim 0.6 - 0.8 M_\odot$, where the flow goes supersonic, and not at the center of the star. As time goes on hot material accretes onto the nascent neutron star and collapses to above nuclear matter density. This accreted material cools via neutrino emission through both pair processes and β–reactions. The entropy is lowered from its initial hot value (S up to $S = 10$ in first tens of msecs; $S \sim 30$ at one second), until when the material reaches nuclear matter density, it has $S \sim 3.5 - 4.0$, is non–degenerate, and passes near the critical point for the liquid–vapor phase transition. The temperature can go up to $T \sim 80$ MeV or even higher; since $\varepsilon_F \sim 38$ MeV, matter is non–degenerate; *i.e.*, $\frac{\pi T}{\varepsilon_F} > 1$. Use of degenerate expressions for the kinetic energy gives *too low* a temperature at a fixed entropy; thus temperatures would be underestimated.

Runs 020–025 and 033 use such degenerate expansions as in earlier work [2, 3, 4, 5, 9] while runs 038–042 use new improved thermal energies in which non–degeneracy is incorporated for the nuclear material [14].

Figure 4. Profiles of Run 022 (solid lines) and Run 023 (dashed lines) at 0.109 seconds after bounce. As a function of $\log_{10} R$ are shown: v_9, the velocity in units of 10^9 cm s^{-1}; S the entropy per nucleon; $\log_{10} \rho$ the cgs density; T the matter temperature in MeV; Y_l the number of leptons per nucleon; and Y_e the number of electrons minus positrons per nucleon.

Figure 5. Same quantities as displayed in Figure 4, for Run 038, except that the abscissa is the Lagrangean mass coordinate. Shown on the same graph are quantities at bounce, and at $T = 0.1,\ 0.2, \ldots 1.1$ seconds after bounce.

Figure 5 shows the profile of Run 038 at times from bounce to 1.1 seconds afterwards. The temperatures gradually rise until at $t = 1.1$ seconds a maximum of $T \sim 75$ MeV is obtained at just about nuclear matter density, with $S \sim 3.5$. The cooling is clearly seen as the entropy spike moves further out in mass with time. Such high temperatures have not been seen in earlier simulations.

7. Neutrino fluxes for the first second

Figure 6. Neutrino fluxes from Run 038. The first graph shows L_{51}, the luminosity at the edge of the computational grid in units of 10^{51} erg s^{-1}. The short–dashed line gives the contribution of ν_e, the long–dashed line that of $\bar{\nu}_e$ and the dashed–dotted line that of each species of $\nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau}$, and the solid line gives the sum of all types. The second figure gives the average neutrino energy for each type of neutrino as it leaves the grid (in MeV).

Continuing with the analysis of Run 038, Figure 6 shows the neutrino fluxes and energies leaving the edge of the grid as a function of time. The hierarchy, $E_{\nu_e} < E_{\bar{\nu}_e} < E_{\nu_\mu}$ which arises because ν_e and $\bar{\nu}_e$ have both charged and neutral current interactions and thus a higher opacity, which means their last scattering surface is farther out than for the $\nu_{\mu,\tau}$ neutrinos and antineutrinos, and thus at lower temperatures. Table 2 shows, however, that while the ν_e and $\bar{\nu}_e$ losses are similar, at 57 and 50×10^{51} ergs, that in the pair neutrinos is much less *per species* at $59.27/4 \sim 15 \times 10^{51}$ ergs; *i.e.*, there is no equipartition of neutrino losses yet at more than a second after bounce even though a total of 166×10^{51} ergs has already left, a significant fraction of the total binding energy of the neutron star to be born. If such a pattern continues much longer, the multiplication factor by which one obtains the total energy release from the antineutrino flux measured on earth from Supernova 1987A will have to be reduced from the value of 6.5 often used.

Run 039 has a somewhat softer EOS than Run 038, with a maximum mass a little too low to satisfy observations, as shown in Table 1, $M_G = 1.313 M_\odot$. It collapses into a black hole at $t = 0.42$ seconds after bounce. Up to that point, however, neutrino fluxes look almost identical to those of Run 038. Subsequent events can not be followed within the present hydrodynamical code due to coordinate singularities, but presumeably the neutrino signal will rapidly die off after the black hole forms.

8. Inclusion of pions

At high temperatures pions will appear. Local equilibrium of reactions such as $p + \pi^- \leftrightarrow n$, $\nu_e + \pi^- \leftrightarrow e^-$, $\nu_e + n \leftrightarrow e^- + p$ implies

$$\mu_{\pi^-} = \mu_n - \mu_p = \mu_e - \mu_{\nu_e} \equiv \hat{\mu} \tag{11}$$

while charge conservation demands the proton fraction must go up. Because the material is very neutron rich we will get appreciable π^- but not π^0 or π^+. In practice π^- appear when $T \gtrsim 20$ MeV and become significant by $T \sim 40$ MeV. In Runs 040, 041 and 042 pions have been included and the results very briefly noted in Table 2. Results are not significantly different except that Run 040 goes into a black hole slightly earlier than Run 039, because the EOS is somewhat softer and the the star bounces at higher density, causing the whole process to proceed somewhat more rapidly.

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